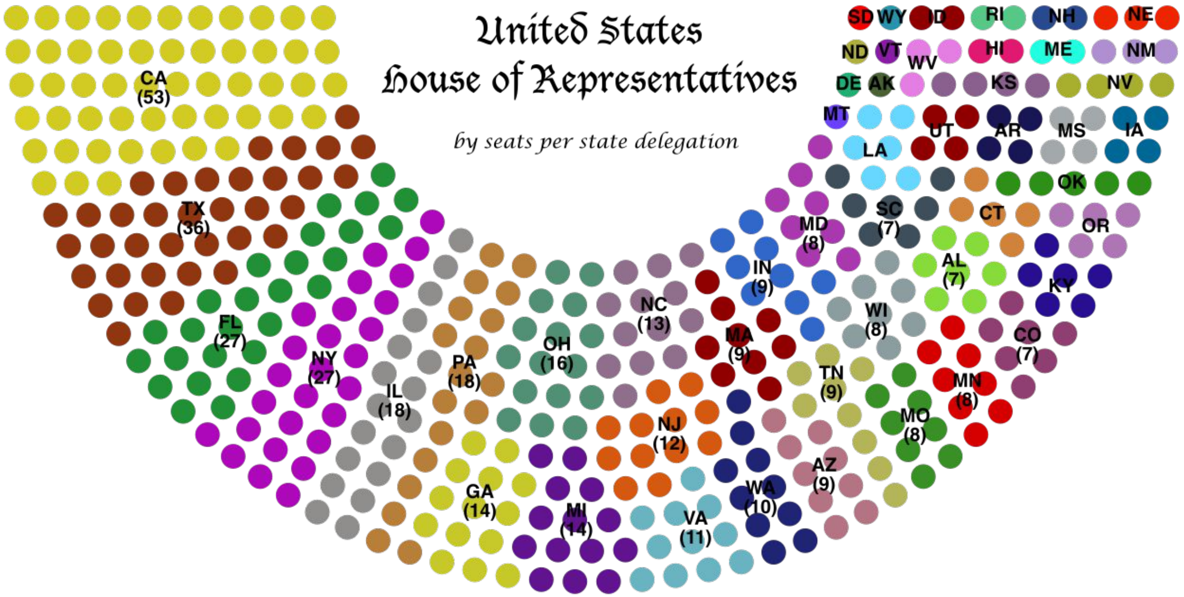


Apportionment

Rebecca Ye ‘22

November 3, 2020

CONGRESSIONAL SEATS

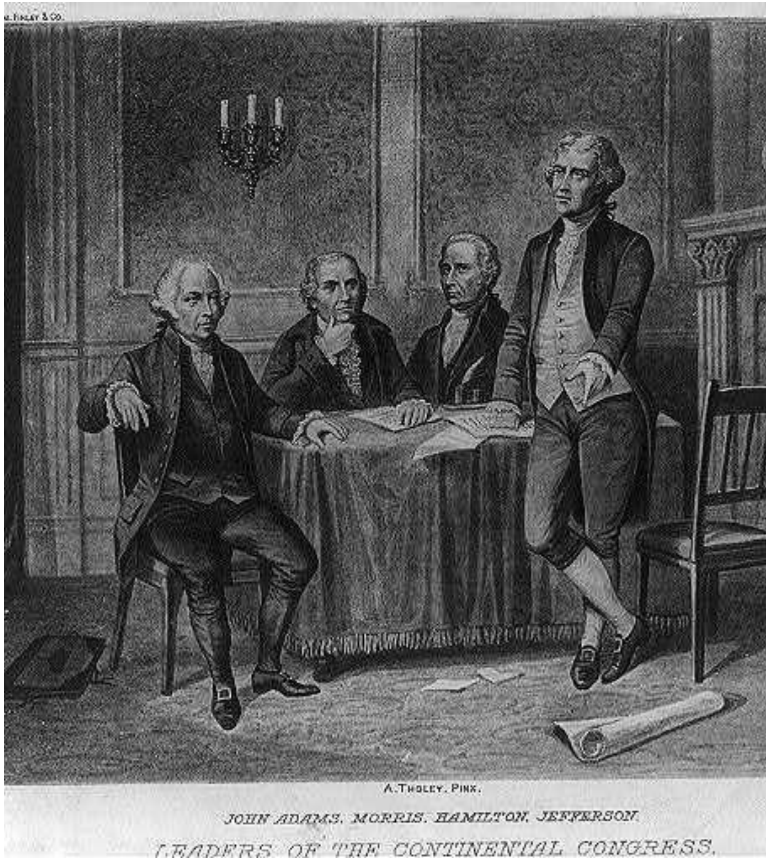
2010
OFFICIAL RESULTS

How do we decide how many seats each state gets?

History

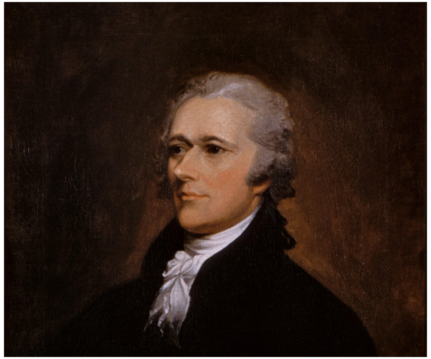
“Representatives shall be apportioned among the several states according to their respective numbers”

- Reapportioned after every census
- Started with 59 seats in 1789
  - 1 rep = 30,000 people
- Locked at 435 seats since 1929
  - Today, 1 rep = 755,000 people



Methods for Apportionment

Hamilton's Method (1852-1911)



Suppose there are 4 states and 20 seats:

Divisor = Total Population/Total Seats = 594.1

Quota = State Population/Divisor, **rounded down**

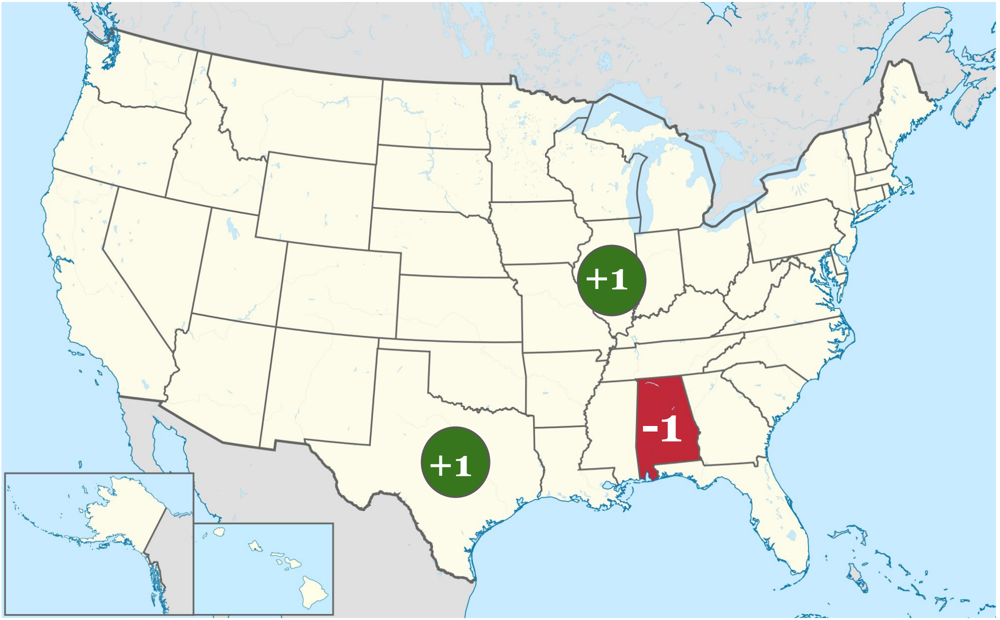
State	Population	Quota	First Allocation of Seats	Leftover Decimal	Seats Apportioned
A	2560	2560/594.1 = 4.31	4	.31	4
B	3315	3315/594.1 = 5.58	5	.58	**6**
C	995	995/594.1 = 1.67	1	.67	**2**
D	5012	5012/594.1 = 8.44	8	.44	8

Alabama Paradox

The Alabama paradox is when an increase in the total seats results in a state losing a seat.

Increasing the house size from 299 to 300 meant Alabama would lose a seat!

House
299 → 300



Population Paradox

A population paradox is when a faster-growing state loses a seat to a slower-growing state.

In 1900, Virginia lost a seat to Maine, even though Virginia's population was growing at a faster rate than Maine's.

Hypothetical population shift: 3 States, 10 Seats

State	Population	Standard Quota	Apportionment	Population	Standard Quota	Lower Quota	Apportionment
A	1,450,000	1.45	2	1,470,000 (+1%)	1.55	1	**1**
B	3,400,000	3.40	3	3,380,000 (-1%)	3.56	3	**4**
C	5,150,000	5.15	5	4,650,000 (-10%)	4.89	4	5
pop = 10,000,000, div = 1,000,000				**pop = 9,500,000, div = 950,000**			

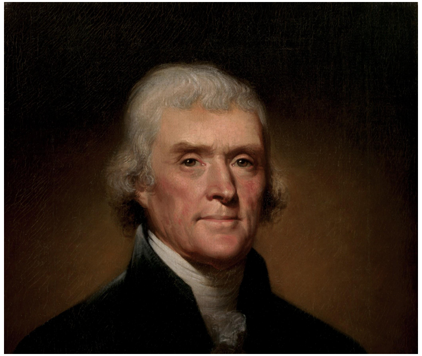
New State Paradox

A new state paradox is when the introduction of a new state results in an existing state losing a seat.

Also known as Oklahoma paradox.

Hamilton's method, 1907					
		BEFORE Oklahoma (386 seats)		AFTER Oklahoma (391 seats)	
State	**Population**	**Quota**	**Apportionment**	**Quota**	**Apportionment**
NY	7,264,183	37.605	38	37.589	**37**
ME	694,466	3.595	3	3.594	**4**

Jefferson's Method (1792-1842)



Lower the divisor until it "fits" the number of seats.

Divisor = Total Population/Total Seats = **594.1**

Now try a lower divisor: **550**

State	Population	Quota	Seats Apportioned
A	2560	2560/**594.1** = 4.31	4
B	3315	3315/**594.1** = 5.58	5
C	995	995/**594.1** = 1.67	1
D	5012	5012/**594.1** = 8.44	8
2 seats left over!			

Quota	Seats Apportioned
2560/**550** = 4.65	4
3315/**550** = 6.03	6
995/**550** = 1.81	1
5012/**550** = 9.11	9
All 20 seats apportioned.	

Failure of Quota Rule

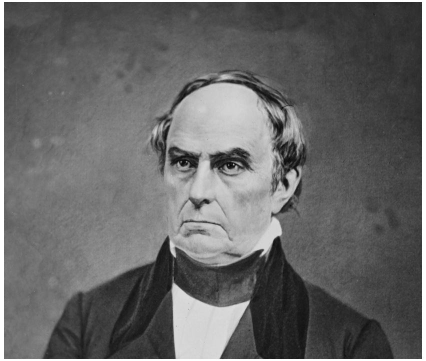
Quota rule = Apportioned seats should lie between the upper and lower roundings.

(ie, when the quota is 5.8, the apportioned seats should be 5 or 6)

State	Population	Standard Quota	Lower Quota	Upper Quota	Hamilton's apportionment	Modified Quota	Jefferson's Apportionment
A	1,500,000	1.5	1	2	2	1.88	1
B	1,400,000	1.4	1	2	1	1.75	1
C	1,300,000	1.3	1	2	1	1.62	1
D	5,800,000	5.8	5	6	6	7.25	7
State D receives more seats than its upper quota.							

Note: Lowered divisor favors **larger** states.

Webster's Method (1842-1852, 1911-1940)



Like Hamilton's method, but uses traditional rounding.

Leftover decimal > 0.5, round **up** **(4.51 → 5)**
Leftover decimal <0.5, round **down** **(4.49 → 4)**

State	Population	Quota	Leftover Decimal	Seats Apportioned
A	2560	2560/594.1 = 4.31	.31	4
B	3315	3315/594.1 = 5.58	.58	**6**
C	995	995/594.1 = 1.67	.67	**2**
D	5012	5012/594.1 = 8.44	.44	8

Note: Can fail the **quota rule**!

Huntington-Hill (1941-present)

Like Webster's, but rounds with geometric mean.

Geometric Mean = √[n * (n+1)] = √[lower*upper]
Quota > Geometric Mean, round **up** **(4.48 → 5)**
Quota < Geometric Mean, round **down** **(4.46 → 4)**

State	Population	Quota	Lower Quota	Upper Quota	Geometric Mean	Seats Apportioned
A	2560	2560/594.1 = 4.31	4	5	4.47	4
B	3315	3315/594.1 = 5.58	5	6	5.48	6
C	995	995/594.1 = 1.67	1	2	1.41	2
D	5012	5012/594.1 = 8.44	8	9	8.49	8

Note: Geometric mean tends to favor **smaller** states over larger ones.

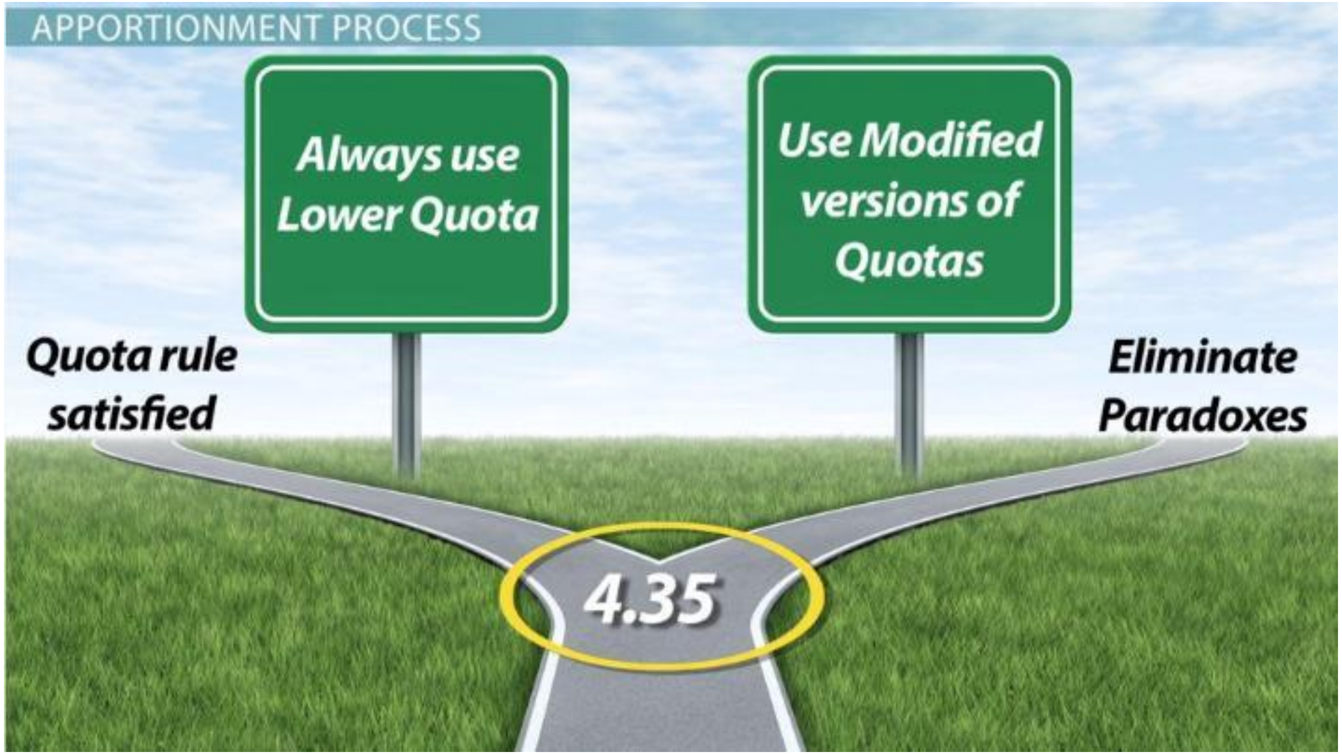
Summary

Method	Quota Rule	Lower Quota Rule	Upper Quota Rule	No Alabama Paradox	No Population Paradox
Hamilton	Yes	Yes	Yes	No	No
Lowndes	Yes	Yes	Yes	No	No
Adams	No	No	Yes	Yes	Yes
Dean	No	No	No	No	Yes
Huntington-Hill	No	No	No	No	Yes
Webster	No	No	No	No	Yes
Jefferson	No	Yes	No	Yes	Yes

No method satisfies “Yes” across all categories.

Balinski-Young Theorem

The "Apportionment paradox"



Balinski-Young Theorem

Theorem (Balinski-Young, 1982)

No neutral apportionment method can satisfy all of the following criteria at once.

1. Quota maintained
2. House monotonicity (no Alabama paradox)
3. Population monotonicity (no population paradox)

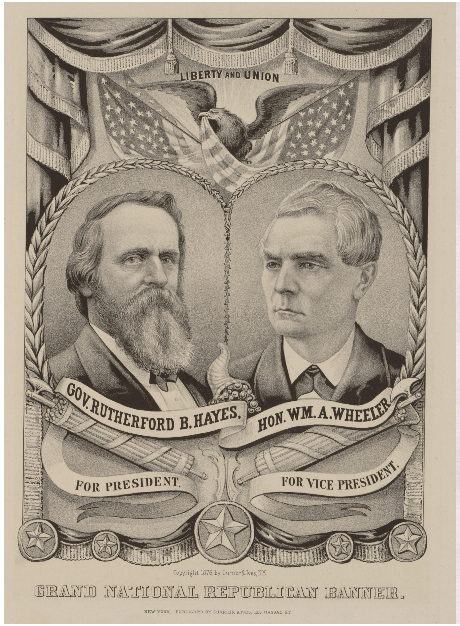
Sounds like… Arrow's Impossibility Theorem? Similar idea.

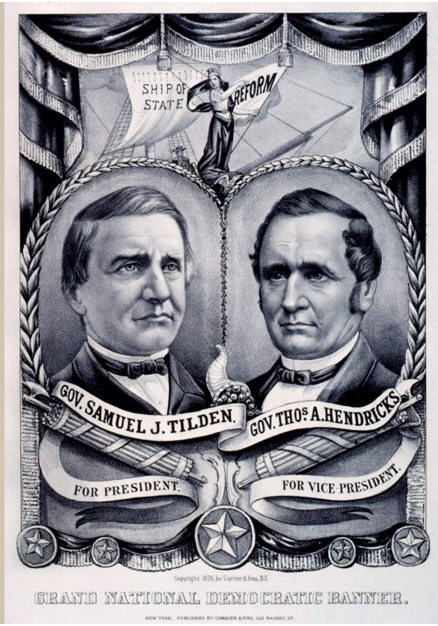
Hayes vs. Tilden, 1876

Rutherford Hayes won with 185 electoral college votes.

Samuel Tilden received 184, and won the popular vote.

Under Webster's method, Tilden would have won over Hayes.





Balinski and Young prefer… Webster?

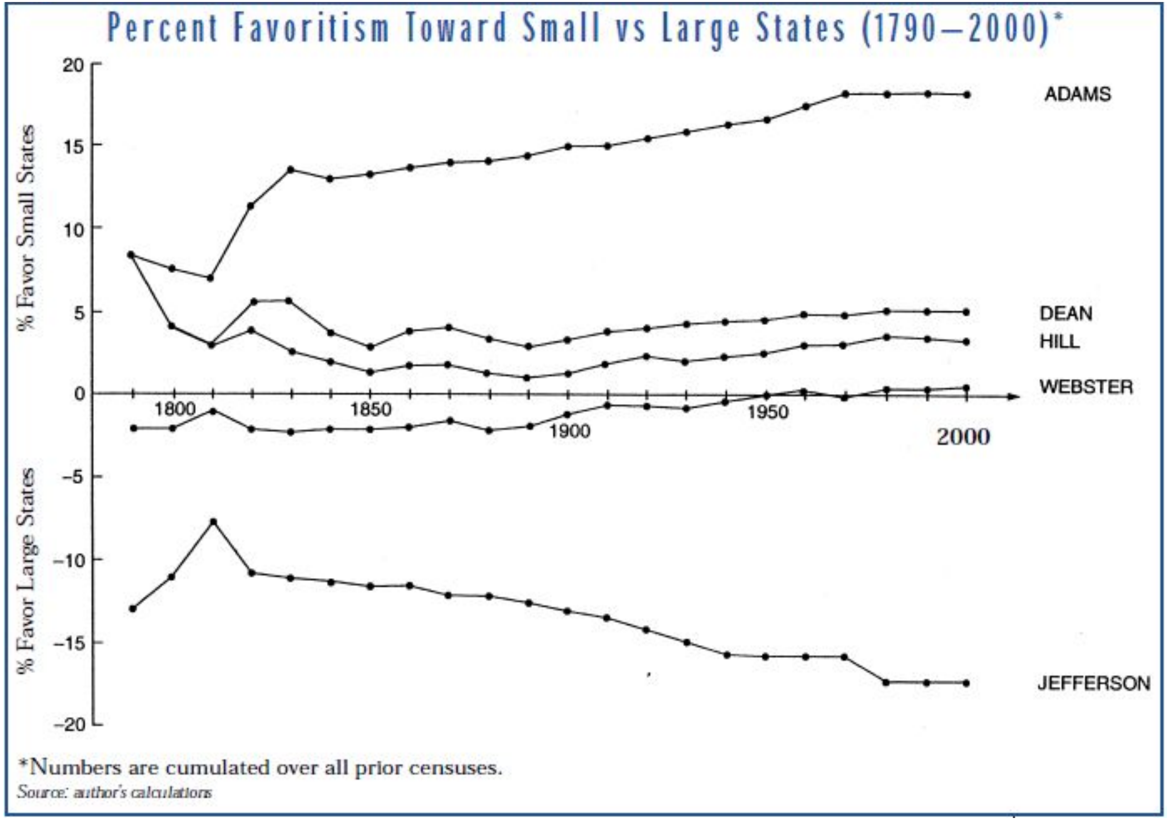


Figure 1. Graph of percent favoritism toward Small vs Large States (1790-2000) from *Dividing the House: Why Congress Should Reinstate an Old Reapportionment Formula* by H. Peyton Young, August 31, 2001. https://www.brookings.edu